Improved Diagonal Loading for Robust Adaptive Beamforming Based On LS-CMA

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Abstract – Least Square-Constant modulus algorithms (LS-CMA) is a popular method for blind adaptive beamforming. LS-CMA is efficient and having good performance when used in communication channels having constant modulus properties like FM, MSK, QPSK, PSK or BPSK. However, LS-CMA can also be used successfully with signals which are non-constant modulus if the Kurtosis of the beam former output is less than two. This is greatly significant because it vindicate that the LS-CMA is also robust to symbol timing error when used to pulse shaped PSK signals. Despite its effectiveness, good performance and its wide applications, LS-CMA comes along with several sensitive problems that have never really been solved. In particular LS-CMA has a tendency of steering the maximum radiation power in the direction of interfering signal even if the interfering signals are not modulated, thus causing interference. In this paper, a general knowledge of blind adaptive beamforming is briefly reviewed to create a good awareness to the readers of this paper and a new concept in diagonal loading is formulated to solve the sited problem. The new concept formulated in this paper involves reconstruction of covariance matrix of the input data where spatial matched filter is used; and its output added diagonally to identity matrix. The simulation results which was done by comparing the developed algorithm with the convectional RLS-CMA, conventional LS-CMA and modified LS-CMA in [1] **significantly confirmed that the proposed algorithm solved the problem and highly improved the performance of adaptive beamforming based on LS-CMA.**

Index Terms – Constant modulus algorithm (CMA), Diagonal loading, Signal-to-interference plus Noise Ratio (SINR), Simulation, LS-CMA, Diagonal Loading-CMA (DL-CMA), Adaptive beamforming.

1. INTRODUCTION

Adaptive beamforming is a concept of signal processing technique which uses an array of sensors to receive signals which are coming from an expected directions and nulls the signals which are coming from unwanted directions. In recent decades the adaptive beamforming algorithms have developed rapidly, with many algorithms proposed, List Square Constant Modulus Algorithm (LS-CMA) is one of many popular algorithms proposed for adaptive beamforming [2]. Despite its effectiveness and its wide use; LS-CMA comes with some challenges and one specific challenge is a problem of capturing interfering signals even if the interfering signals are not modulated. In this paper, the improved diagonal loading technique is proposed to solve the problem of steering the maximum radiation power in the direction of interfering signal even if the interfering signals are not modulated [2].

The main principle operation of the adaptive beam forming is to steer main beam at the direction of the desired signal and at the same time to null the signals from unwanted directions. This is achieved by adjusting weights in proportion to the signal samples, the weight adjusted until the appropriate weight obtained which can enable the maximum gain to be directed to the signal of interest and minimum gain to be directed to the interferences. Most of the methods about adaptive beamforming are introduced to meet some iterative performance criteria such as Maximum signal-to-interference plus noise ratio (SINR)[3], Minimum Mean Square Error (MMSE), Minimum noise variance, maximum likelihood (ML) and Maximum gain[4].

The adaptive beamforming algorithms when applied to the system like mobile wireless communication instead of conventional antenna system can minimize the problem of multipath fading, co-channel interference and background noise. Also adaptive beamforming can achieve a better quality of service, improve geographical coverage area of the service and increasing the capacity of the service.

Adaptive beamforming has been introduced for the so many decades and has become more useful in many applications such as wireless communications, radar, astronomy, seismology, medical-imaging, acoustics, and sonar[5][6][7][8][9].

1.1. Adaptive beamforming algorithms

Basically, there are two (2) classes of Adaptive beamforming algorithms: Blind adaptive beamforming and, Non-blind adaptive beamforming

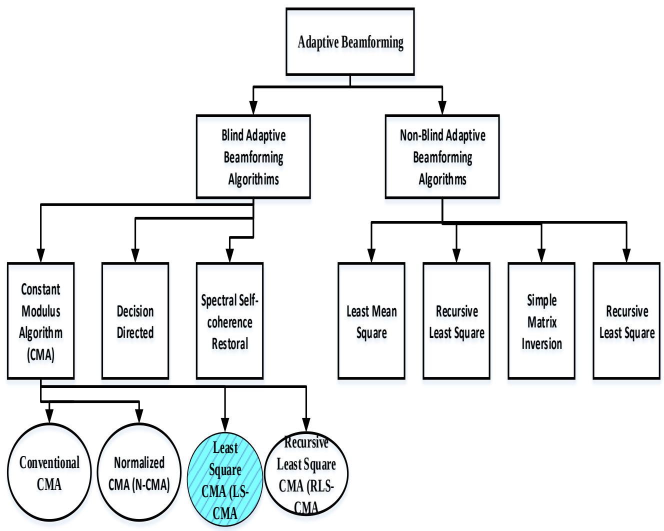

Figure 1 Classes of beamforming algorithms

1.2. Blind adaptive beamforming algorithms

Blind adaptive beamforming algorithms do not use a reference (trained) signal or other special information about the environment to adjust (update) the weights. Blind adaptive beamforming have several types including; Constant modulus algorithm (CMA), Spectral Self-Coherence Restoral (SCORE) and, Decision Directed algorithm (DD).

1.3. Non- blind adaptive beamforming algorithms

Non-blind algorithms use a reference (trained) signal so as to detect the desired signal, also use a reference (trained) signal to adjust (update) the weights. Non-blind adaptive beamforming have several types including; Least mean squares (LMS), Recursive least squares (RLS), Simple matrix inversion (SMI) and, Conjugate gradient method (CGM).

1.4. The basics of blind adaptive beamforming

This part represents the general review of some abstract knowledge and methods used in the blind adaptive beamforming intending to provide a brief review of the blind adaptive beamforming techniques. Signal model, conventional CMA and, conventional LS-CMA will be briefly discussed in this part[10].

Signal model: The signal model has some basic parts and terms as briefly described below. The signal model parts or items include; Narrowband model, Array factor and, Antenna array beam pattern

a. Narrowband model:

The narrowband model is very important part of signal model because it is used to identify the delay of the signals received by one antenna relative to another antenna. The antennas generally arranged in an array with the same distance interval between them. In this model the simple carrier phase shift takes the place of time delay.

In this case, the blind adaptive beamforming technique consists of $\boldsymbol{L}$ number of sensor elements. The sensor elements arranged in the uniform linear array (ULA)[11] and have a distance $d = {}^{\lambda}/_{2}$ between each sensor elements. The signals to be received are $M+1$ narrow band signals with a centre frequency (f_c). (One desired signal and M interference signals). Also $M+1$ should be less than L. $\boldsymbol{(M+1<L)}$. See the figure below;

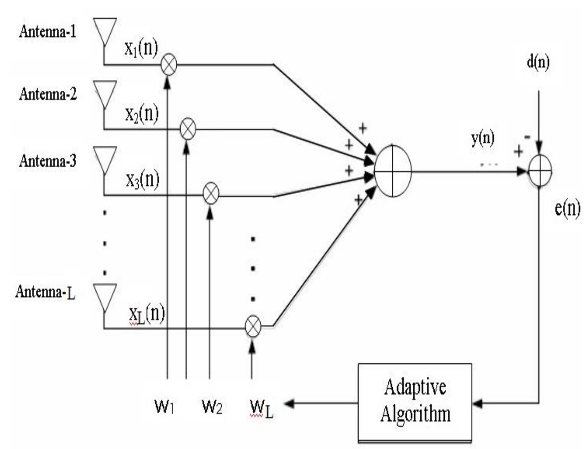

Figure 2 Uniform Linear Array (ULA) diagram

The signals to be received by antenna given as;

$$\boldsymbol{S_d}(t) = \boldsymbol{V_d}(t)e^{j2\pi f_c t} \quad (1)$$

$$\boldsymbol{S}(t) = \boldsymbol{V_i}(t)e^{j2\pi f_c t} \quad (2)$$

Where $\boldsymbol{S_d}(t)$ and $\boldsymbol{S}(t)$ are desired signal and interference signal respectively also $\boldsymbol{V_d}(t)$ and $\boldsymbol{V_i}(t)$ are the amplitude of desired signal and interfering signal.

The signal to be received by the first antenna given as;

$$\begin{aligned}\boldsymbol{x_1}(t) &= \boldsymbol{S_d}(t) + \boldsymbol{S}(t) + \boldsymbol{N_o}(t) \quad (3)\\ &= \boldsymbol{V_d}(t)\, e^{j2\pi f_c t} + \boldsymbol{V_i}(t)e^{j2\pi f_c t} + \boldsymbol{N_o}(t)\\ &= [\boldsymbol{V_d}(t)\ + \boldsymbol{V_i}(t)]e^{j2\pi f_c t} + \boldsymbol{N_o}(t)\end{aligned}$$

Where: $\boldsymbol{N_o}(t)$ denotes the white Gaussian noise.

The signal received at the Lth antennas given as:

$$\begin{aligned}\boldsymbol{X_L}(t) &= \boldsymbol{S_d}(t+\tau_L) + \boldsymbol{S}(t+\tau_L) + \boldsymbol{N_o}(t)\\ &= [\boldsymbol{V_d}(t) + \boldsymbol{V_i}(t))]e^{j2\pi f_c(t+\tau_L)} + \boldsymbol{N_o}(t)\\ &= [\boldsymbol{V_d}(t) + \boldsymbol{V_i}(t)+)]e^{j2\pi f_c t}e^{j2\pi f_c \tau_L} + \boldsymbol{N_o}(t)\\ &= [\boldsymbol{S_d}(t) + \boldsymbol{S}(t)\ +]e^{j2\pi f_c \tau_L} + \boldsymbol{N_o}(t) \quad (4)\end{aligned}$$

As seen above, the array factors given as: $\boldsymbol{a}(\theta) = e^{j2\pi f_c \tau_L}$ and $\boldsymbol{a}(\theta_d) = e^{j2\pi f_c \tau_L}$

Where: τ_L is the time taken for the signal arrived in each antenna; normally this time is called the time difference of arrival (TDOA).

But the term $e^{j2\pi f_c \tau_L}$ is normally called the Array factor. The array factors given as:

$$\boldsymbol{a}(\theta) = e^{j2\pi f_c \tau_L} \tag{5}$$

$$\boldsymbol{a}(\theta_d) = e^{j2\pi f_c \tau_L} \tag{6}$$

Where;

$\boldsymbol{a}(\theta)$ Is an array factor for interference signals.

$\boldsymbol{a}(\theta_d)$ Is an array factor for desired signal.

Therefore the signal received at the Lth antennas given as:

$$\boldsymbol{X_L}(t) = \boldsymbol{a}(\theta_d)\boldsymbol{S_d}(t) + \boldsymbol{a}(\theta)\boldsymbol{S}(t) + \boldsymbol{N_o}(t) \tag{7}$$

a. Array factor:

Array factor is used to explain the change of phase and amplitude of the signal when propagate from one side to another side of the antenna array. Array response vector is significant in this model because the system deals with multidimensional data so it is better to work with data in vector representation.

$$\tau_L = \frac{d \sin\theta}{c} \tag{8}$$

Where θ is the angle of arrival **(DOA)** of the signal and $\boldsymbol{c}$ is the propagation speed of the signal

Therefore, we get;

$$\boldsymbol{a}(\theta) = e^{j2\pi f_c\left(\frac{d\sin\theta}{c}\right)} \tag{9}$$

$$\boldsymbol{a}(\theta_d) = e^{j2\pi f_c\left(\frac{d\sin\theta}{c}\right)} \tag{10}$$

The action of finding the array factor (Array response vector) in term of θ is called array calibration. The n discrete instant time also called the snapshot of the data signal, for the only one interference signal received can be given as;

$$\boldsymbol{X}(\boldsymbol{n}) = \boldsymbol{a}(\theta_d)\boldsymbol{S_d}(n) + \boldsymbol{a}(\theta)\boldsymbol{S}(n) + \boldsymbol{N_o}(n) \tag{11}$$

For the M incident interference signals, the signal received at an Array of antenna given as;

$$\boldsymbol{X}(n) = \boldsymbol{a}(\theta_d)\boldsymbol{S_d}(n) + \sum_{\boldsymbol{j=1}}^{\boldsymbol{M}} \boldsymbol{a}(\theta_j)\boldsymbol{S_j}(n) + \boldsymbol{N_o}(n) \tag{12}$$

Where the size of $\boldsymbol{X}(n)$ is $L \times N$, and N is the number of snapshot

C. Antenna array beam patterns:

The signal received at any array of an antenna can be weighted and summed to improve the quality of the desired signal. This can be expressed mathematically as:-

$$\boldsymbol{y}(n) = \sum_{i=1}^{N} \boldsymbol{w^*}_i \boldsymbol{x}_i(n) \tag{13}$$

Where: $\boldsymbol{w^*_i}$ is the weight applied to the received data at the ith sensors. By using the matrix notation, the antenna array output given as:-

$$\boldsymbol{y}(n) = \boldsymbol{w}^H\boldsymbol{X}(n) \tag{14}$$

Where $\boldsymbol{w}$ is the $L \times 1$ complex vector of antenna. The output array expressed as;

$$\boldsymbol{y}(n) = \boldsymbol{w}^H\boldsymbol{a}(\theta_d)\boldsymbol{S_d}(n) + \sum_{j=1}^{M} \boldsymbol{w}^H\boldsymbol{a}(\theta_j)\,\boldsymbol{S}_j(n) + \boldsymbol{w}^H\boldsymbol{N_o}(n) \tag{15}$$

The suppression and passing of the signal from the angle-of-arrival (AOA) is determined by using the weight vector ($\boldsymbol{w}$) and array response vector.

The beam pattern is used to show the gain versus the angle-of-arrival (AOA). The beam pattern expressed mathematically as;

$$\boldsymbol{F}(\theta) = |\boldsymbol{w}^H\boldsymbol{a}(\theta)|^2 \tag{16}$$

1.5. Conventional Constant Modulus Algorithm (CMA) for Adaptive beam forming

Constant modulus algorithms (CMAs) are the most popular in blind adaptive beamforming algorithms and are the most useful techniques in the radar, astronomy, seismology, medical-imaging, acoustics, sonar, and wireless communications research and applications[12][13].

This algorithm has a good efficiency and good performance if the signals in the communication channels have constant modulus properties like FM, MSK, QPSK, PSK or BPSK [14]. One of the mostly characteristic properties of the CMA is that carrier synchronization is not compulsory [15][16]. Besides, it can be used successfully to the signal which is non-constant modulus if the Kurtosis of the beam former output is less than two. That means, CMA can be used to the PSK signals which have non-rectangular pulse shape. This is great significance because it vindicate that the CMA is also robust to symbol timing error when used to pulse shaped PSK signals. Pulse shaping usually is used to limit the transmitted signal bandwidth being used. This technique is Stochastic Gradient Descent (SGD) method with the intension to make the cost function very minimum, $J(a, b)$ given by;

$$J_{(a,b)} = [|\boldsymbol{y}(n)|^a - |\alpha|^a]^b \quad (17)$$

Where; $\boldsymbol{y}(n) = \boldsymbol{w}^H\boldsymbol{X}(n)$ is the output estimated desired signal. The values of "a" and "b" are careful chosen to produce a different types of CMA algorithm. Usually CMA algorithms are categorized as (a, b) $-CMA$. Therefore by setting the value of $a = 1$, $b = 2$ and $\alpha = 1$, it can be expressed as;

$$J_{(1,2)} = [|\boldsymbol{y}(n)| - 1]^2 \quad (18)$$

The expectation of cost function given as; $E\{J_{(1,2)}\} = E\{[|\boldsymbol{y}(n)| - 1]^2\}$

Applying the gradient that minimize the cost function of the CMA algorithm

$$\Delta w(J_{(1,2)}) = 2\frac{\partial[E\{J(n)\}]}{\partial w}$$

$$= 2(|\boldsymbol{y}(n)| - 1)\boldsymbol{X}(n)\frac{\boldsymbol{y}(n)}{|\boldsymbol{y}(n)|} \quad (19)$$

Therefore the expression for updating the weight is given by;

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + 2\mu\nabla\boldsymbol{w}(J_{(1,2)})$$

$$= \boldsymbol{w}(n) + 2\mu\left(\frac{\boldsymbol{y}(n)}{|\boldsymbol{y}(n)|} - \boldsymbol{y}(n)\right)\boldsymbol{X}^H(n) \quad (20)$$

Where: $\frac{\boldsymbol{y}(n)}{|\boldsymbol{y}(n)|}$ is the hard limit expressed as $\boldsymbol{d}(n)$ and μ is the step size.

Therefore: $\frac{\boldsymbol{y}(n)}{|\boldsymbol{y}(n)|} - \boldsymbol{y}(\boldsymbol{n}) = \boldsymbol{d}(n) - \boldsymbol{y}(n) = \boldsymbol{e}(n)$

Moreover, it can be applied to non-constant modulus signals which have the kurtosi less than two. The kurtosis, k_y expressed as:

$$k_y = \frac{[|\boldsymbol{y}(n)|]^4}{[|\boldsymbol{y}(n)|^2]^2} \quad (21)$$

From the above expression, the constant envelope/magnitude of the signals have a kurtosis of one. Complex Gaussian noise particularly has kurtosis of two, the signal to noise ratio (SNR) of constant modulus signal received in Gaussian noisy data is expressed as:

$$SNR = \frac{2 - k_y + \sqrt{2 - k_y}}{k_y - 1} \quad (22)$$

Therefore, ask_y approches to 1, SNR approches to infinity, and as, k_y approches to 2, SNR approches to zero. Thus CMA algorithm can be applied to the signals which have kurtosis less than two.

Stochastic Gradient Descent (SGD) CMA has a slow convergence rate therefore modified methods were developed as shown on the preceding subsections.

1.6. Conventional Least Square Constant Modulus Algorithm (Conventional LS-CMA)

The cost function of this method is given as; $J_{(a,b)} = [|\boldsymbol{y}(n)|^a - |c|^a]^b$

Where: c is the desired signal amplitude at the output array. The signal used has the constant envelope;

Therefore the amplitude of the desired signal is 1. The cost function becomes as; $J_{(a,b)} = [|\boldsymbol{y}(n)|^a - 1]^b$

$$J_{(1,2)} = J(w) = [|\boldsymbol{y}(n)| - 1]^2 = \|h_k(w)\|_2^2 = \|h_k(w)\|^2 \quad (23)$$

Consider *nth* signals, *nth* signal is a nonlinear function given by | $\boldsymbol{h_k}(\boldsymbol{w})$|

$$\boldsymbol{h_k}(\boldsymbol{w}) = [h_1(\boldsymbol{w}),\ h_2(\boldsymbol{w}),\ h_3(\boldsymbol{w}),\ h_4(\boldsymbol{w}) \dots\dots h_K(\boldsymbol{w})]^T \quad (24)$$

Applying the square to the cost function we get partial Taylor series expressed as;

$$J(\boldsymbol{w} + \boldsymbol{z}) = \|\boldsymbol{h}(\boldsymbol{w}) + \boldsymbol{D}^H(\boldsymbol{w})\boldsymbol{z}\|_2^2 \quad (25)$$

Where: $\boldsymbol{z}$ is the offset vector and $\boldsymbol{D}(\boldsymbol{w})$ is given as;

$$\boldsymbol{D}(\boldsymbol{w}) = \nabla\boldsymbol{h_w}(\boldsymbol{w}) = [\nabla(h_1(\boldsymbol{w})),\ \nabla(h_2(\boldsymbol{w})),\ \nabla(h_3(\boldsymbol{w})),\ \nabla(h_4(\boldsymbol{w})) \dots\dots \nabla(h_K(\boldsymbol{w}))]$$

The gradient of the cost function is given by;

$$\nabla J(\boldsymbol{w} + \boldsymbol{z}) = 2\frac{\partial J(\boldsymbol{w} + \boldsymbol{z})}{\partial \boldsymbol{z}^*} \quad (26)$$

$$\nabla J(\boldsymbol{w} + \boldsymbol{z}) = 2\frac{\partial\{[\boldsymbol{h}(\boldsymbol{w}) + \boldsymbol{D}^H(\boldsymbol{w})\boldsymbol{z}]^H[\boldsymbol{h}(\boldsymbol{w}) + \boldsymbol{D}^H(\boldsymbol{w})\boldsymbol{z}]\}}{\partial \boldsymbol{z}^*} \quad (27)$$

$$\nabla J(\boldsymbol{w} + \boldsymbol{z}) = \boldsymbol{2}\frac{\partial\{\|\boldsymbol{h}(\boldsymbol{w})\|_2^2 + \boldsymbol{h}^H(\boldsymbol{w})\boldsymbol{D}^H(\boldsymbol{w})\boldsymbol{z} + \boldsymbol{z}^H\boldsymbol{D}(\boldsymbol{w})\boldsymbol{h}(\boldsymbol{w}) + \boldsymbol{z}^H\boldsymbol{D}(\boldsymbol{w})\boldsymbol{D}^H(\boldsymbol{w})\boldsymbol{z}\}}{\partial \boldsymbol{z}^*} \quad (28)$$

Where: the symbol (*) represent the conjugate.

$$\nabla J(\boldsymbol{w} + \boldsymbol{z}) = 2[\boldsymbol{D}(\boldsymbol{w})\boldsymbol{h}(\boldsymbol{w}) + \boldsymbol{D}(\boldsymbol{w})\boldsymbol{D}^{\boldsymbol{H}}(\boldsymbol{w})\boldsymbol{z}] \quad (29)$$

Make the equation $2[\boldsymbol{D}(\boldsymbol{w})\boldsymbol{h}(\boldsymbol{w}) + \boldsymbol{D}(\boldsymbol{w})\boldsymbol{D}^H(\boldsymbol{w})\boldsymbol{z}]$=0, the offset vector $\boldsymbol{z}$ given as;

$$\boldsymbol{z} = -[\boldsymbol{D}(\boldsymbol{w})\boldsymbol{D}^H(\boldsymbol{w})]^{-1}\boldsymbol{D}(\boldsymbol{w})\boldsymbol{h}(\boldsymbol{w}) \qquad (30)$$

The new weight vector $\boldsymbol{w}(n+1)$ that minimize the cost function can be obtained by adding the weight $\boldsymbol{w}(n)$ and offset vector $\boldsymbol{z}$. The new weight vector $\boldsymbol{w}(n+1)$ become:

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \boldsymbol{z} \qquad (31)$$

$$\boldsymbol{w}(n+1)$$

$$= \boldsymbol{w}(n) - \left[\boldsymbol{D}(\boldsymbol{w}(k))\boldsymbol{D}^H(\boldsymbol{w}(k))\right]^{-1}\boldsymbol{D}(\boldsymbol{w}(k))\boldsymbol{h}(\boldsymbol{w}(k))$$

$$= \boldsymbol{w}(n) - (\boldsymbol{X}\boldsymbol{X}^H)^{-1}\boldsymbol{X}\boldsymbol{X}^H\boldsymbol{w}(k) - (\boldsymbol{X}\boldsymbol{X}^{\boldsymbol{H}})^{-1}\boldsymbol{X}\boldsymbol{d}^*(n)$$

$$\boldsymbol{w}(n+1) = (\boldsymbol{X}\boldsymbol{X}^H)^{-1}\boldsymbol{X}\boldsymbol{d}^*(n) \qquad (32)$$

Where; $\boldsymbol{X} = [x(1), x(2), x(3), x(4), \dots \dots \dots . x(n)]^T$ and $\boldsymbol{d}(n)$ is the hard limiter of $\boldsymbol{y}$(n)

But $\boldsymbol{y}(n) = [\boldsymbol{w}^H(k)\boldsymbol{X}]^T$

$$\boldsymbol{d}(n)$$

$$= \left[\frac{y(1)}{|y(1)|}, \frac{y(2)}{|y(2)|}, \frac{y(3)}{|y(3)|}, \frac{y(4)}{|y(4)|}, \dots \dots \dots \frac{y(n)}{|y(n)|}\right]^T \qquad (33)$$

1.7. Our contribution and organization of this paper

This paper presents systematic information which preview and examine the essential knowledge of: adaptive beamforming based on CMA; the adaptive beamforming based on LS-CMA and the proposed algorithm for solving the problem of LS-CMA which is steering the maximum radiation power in the direction of interfering signal even if the interfering signals are not modulated.

The rest of the paper is organized as follows: Section two (2) describes related works done by other researchers, Section three (3) describes the proposed “improved diagonal loading technique for adaptive beamforming”. In Section three (4) simulation results are analyzed and discussed. Section five (5) concludes the paper by summarizing the authors’ views on how the proposed system have improved the performance of LS-CMA and significantly solved the problem of interference.

2. RELATED WORK

Many algorithms have been proposed concerning blind adaptive beamforming. One of the most common blind adaptive beamforming techniques is the conventional constant modulus algorithm (CMA). This algorithm is based on modulated signals having a constant magnitude (modulus) like FM, QPSK, BPSK, QAM MSK etc. The operation performance of the adaptive beamforming is affected by the effects associated with array structure e.g. steering vector errors [2] [3], effect of mutual coupling[15],[16], and channel response errors[17],[18]. In this paper, blind beamforming based on LS-CMA is considered. The quality of LS-CMA and CMA in general is dependent on the initialization of the weight vector and, also the signal-to-interference ratio (SIR). A number of methods have been put forward to achieve good performance of CMA e.g. LS-CMA[2][19], N-CMA [20] and other modified LS-CMA methods[1]. Most of these methods i.e., [2], [19], [20]and [11][1] have a low convergence rate due to inappropriate selection of the initial weight. On the other side, most of the methods subjected from the effect signal self-cancellation. This is due to the power of desired signal being much greater than interference power which results into beam-pattern distortion. Note that with CMA beamforming, when the interference signal also satisfies the constant-modulus property and the power of the desired signal is small compared to power of interference signal, the algorithm can also result in signal self-cancellation thus steering maximum power onto the high power interference signal[1]. In wireless communication like mobile communication is very easy for the interference signal to be a constant envelope modulated signals. These high power interfering signals which are constant envelope modulated signals cause constant modulus algorithms to misbehave from his normal operation and make them to detect the interfering signals as the desired signal and finally the algorithm suppress the signal of interest. One of a specific problem of LS-CMA is the dilemma of capturing the interference signals even if the interfering signals are not modulated signals[2].

The innovation for constant modulus methods are needed, and one being to solve the problem of LS-CMA which is: steering the maximum radiation power in the direction of interfering signal even if the interfering signals are not modulated. Apart from solving the problem the proposed algorithms provides good performance, fast convergence rate, easy to implement by introducing a best way of initialization which is not complex as in [1]

3. IMPROVED DIAGONAL LOADING FOR ROBUST ADAPTIVE BEAMFORMING BASED ON LS-CMA

This part proposes a new concept in diagonal loading to improve the performance of the conventional LS-CMA. This algorithm is developed to overcome the problem of existing LS-CMA which is: steering the maximum radiation power in the direction of interfering signals even if the interfering signals are not modulated.

3.1. Method principle

The proposed method involves reconstruction of covariance matrix of the input data where spatial matched filter is used and its output added diagonally to identity matrix. The blind adaptive beamforming follow the steps in the figure below;

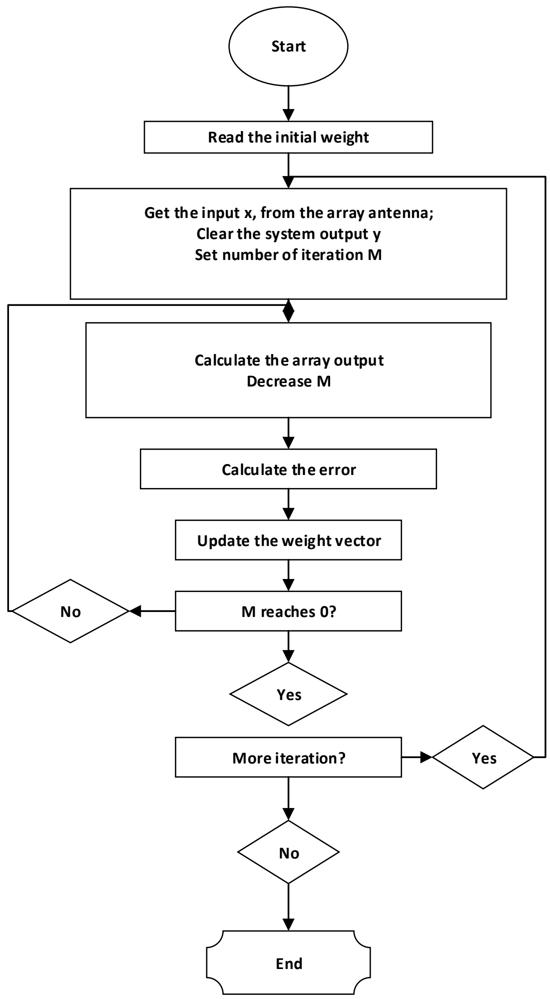

Figure 3 Flow diagram of blind adaptive beamforming

3.2. Implementation of diagonal loading constant modulus algorithm (DL-CMA)

The algorithm implemented by the following steps;

i. Estimate the output signal of the beamformer. The estimation of the output signal is given as;

$$\boldsymbol{y}(n) = \boldsymbol{w}^{\boldsymbol{H}}(n)\boldsymbol{X}(1{:}\,n) \tag{34}$$

The length of $\boldsymbol{y}(n)$ is $1 \times N$. Where N is the number of snapshots.

ii. Compute the hard limit.

The hard limit is given as; $\boldsymbol{d}(n) = \frac{y(n)}{|y(n)|} = \left[\frac{y(1)}{|y(1)|}, \frac{y(2)}{|y(2)|}, \dots \dots \dots \dots \frac{y(N)}{|y(N)|}\right]$

The length of $\boldsymbol{d}(n)$ is $1 \times N$

iii. Compute the estimation error of the beamformer.

The error estimate is given as; $\boldsymbol{e}(n) = \boldsymbol{d}(n) - \boldsymbol{y}(n)$

The size of $\boldsymbol{e}(n)$ is $1 \times N$.

iv. Compute the covariance matrix of the signals. The covariance matrix is given as;

$$\boldsymbol{R_x} = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{X_i}(n)\boldsymbol{X}_i^{\ H}(n) \tag{35}$$

The size of the $\boldsymbol{R_x}$ is $L \times L$. Where L is the number of elements.

v. Normalizing the nominal steering vector of the desired signal as follows;

$$\overline{\boldsymbol{a}(\theta_d)} = \frac{\boldsymbol{a}(\theta_d)}{\|\boldsymbol{a}(\theta_d)\|_2} \tag{36}$$

Where the size of the $\overline{\boldsymbol{a}(\theta_d)}$ is $1 \times L$

vi. Compute the diagonal loading coefficient. The diagonal loading coefficient is given as;

$$\rho = \overline{\boldsymbol{a}(\theta_d)}\, \boldsymbol{R_x}\left(\,\overline{\boldsymbol{a}(\theta_d)}\,\right)^T \tag{37}$$

Where the size of the ρ is 1×1

vii. Reconstruct the covariance matrix as follows;

$$\overline{\boldsymbol{R_x}} = \boldsymbol{R_x} + \rho\boldsymbol{I} \tag{38}$$

Where the size of the $\overline{\boldsymbol{R_x}}$ and $\boldsymbol{I}$ are $L \times L$ **.**

viii. Computing the weight vector of the beamformer. The weight vector is given as follows;

$$\overline{\boldsymbol{w}}(n) = \overline{\boldsymbol{R_x}}^{-1}\boldsymbol{X}(1{:}\,n)\boldsymbol{d}^H \tag{39}$$

Where the size of $\overline{\boldsymbol{w}}(n)$ is $L \times 1$

ix. Updating the weight vector is given as follows;

$\boldsymbol{w}(n+1) = \overline{\boldsymbol{w}}(n)$ **(40)**

3.3. Performance analysis

The adaptive beamforming performance can be described into two measurable characteristic which are: array pattern and array gain. The array pattern can be described into performance measurable characteristic of spatial distribution of electromagnetic field and antenna array energy. The array pattern given as:

$$\boldsymbol{Bw}(\theta) = \boldsymbol{w}^{\boldsymbol{H}}\boldsymbol{a_r}(\theta_r) \tag{41}$$

Where: $\boldsymbol{a_r}(\theta_r)$ is the array response and θ_r is the range of the direction of arrival (DOA)

SINR is described as the measure of the power of a specific wanted signal divided by the total sum of the interference signals power and background noise power. As the power of background noise became very small approximately equal to zero, then we can state SINR as the signal-to-interference ratio

(SIR). Normally we can obtain two SINR, the first SNR is the input SINR ($SINR_{in}$) and the second SINR is the output SINR($SINR_{out}$). Mathematically these SINR given as follows;

$$SINR_{in} = \frac{P_{d\_in}}{P_{i\_in} + P_{n\_in}} \quad (42)$$

$$SINR_{out} = \frac{P_{d\_out}}{P_{i\_out} + P_{n\_out}} \quad (43)$$

Where:

- P_{d_out} : Represents the output desired signal power
- P_{i_out} : Represents the output interference signal power
- P_{n_out} : Represents the output noise power
- P_{d_in} : Represents the input desired signal power
- P_{i_in} : Represents the input interference signal power
- P_{n_in} : Represents the input noise power

Consider the input signal power of the beamformer; the input signal power is given as:

$$P_{in} = |\boldsymbol{X}(n)|^2 = E[\boldsymbol{X}(n)\boldsymbol{X}^H(n)] \quad (44)$$

But $\boldsymbol{X}(n) = \boldsymbol{a}(\theta_d)\boldsymbol{S_d}(n) + \sum_{j=1}^{M} \boldsymbol{a}(\theta_j)\boldsymbol{S}_j(n) + \boldsymbol{N_o}(n)$, therefore we get;

$$P_{in} = E\left[\left(\boldsymbol{a}(\theta_d)\boldsymbol{S_d}(n) + \sum_{j=1}^{M} \boldsymbol{a}(\theta_j)\boldsymbol{S}_j(n) + \boldsymbol{N_o}(n)\right)\left(\boldsymbol{a}(\theta_d)\boldsymbol{S_d}(n) + \sum_{j=1}^{M} \boldsymbol{a}(\theta_j)\boldsymbol{S}_j(n) + \boldsymbol{N_o}(n)\right)^H\right]$$

$$= \left(\boldsymbol{S_d}(n)\right)^2 \boldsymbol{a}(\theta_d)\boldsymbol{a}^H(\theta_d) + \sum_{j=1}^{M} \left(\boldsymbol{S}_j(n)\right)^2 \boldsymbol{a}(\theta_j)\boldsymbol{a}^H(\theta_j) + \boldsymbol{N_o}(n)\boldsymbol{N_o}^H(n)$$

Where:

- $\left(\boldsymbol{S_d}(n)\right)^2 \boldsymbol{a}(\theta_d)\boldsymbol{a}^H(\theta_d)$ is input desired signal power (P_{d_in})
- $\sum_{j=1}^{M} \left(\boldsymbol{S}_j(n)\right)^2 \boldsymbol{a}(\theta_j)\boldsymbol{a}^H(\theta_j)$ is input interference signal power(P_{i_in})
- $\boldsymbol{N_o}(n)\boldsymbol{N_o}^H(n)$is the input noise power (P_{n_in})

Therefore, we get:

$$SINR_{in} = \frac{\left(\left(\boldsymbol{S_d}(n)\right)^2 \boldsymbol{a}(\theta_d)\boldsymbol{a}^H(\theta_d)\right)}{\sum_{j=1}^{M} \left(\boldsymbol{S}_j(n)\right)^2 \boldsymbol{a}(\theta_j)\boldsymbol{a}^H(\theta_j) + \boldsymbol{N_o}(n)\boldsymbol{N_o}^H(n)} \quad (45)$$

Where:

- $\left(\boldsymbol{S_d}(n)\right)^2 \boldsymbol{a}(\theta_d)\boldsymbol{a}^H(\theta_d)$ is the desired signal covariance matrix ($\boldsymbol{R_d}$)
- $\boldsymbol{N_o}(n)\boldsymbol{N_o}^H(n)$ is the noise covariance matrix ($\boldsymbol{R_n}$)
- $\sum_{j=1}^{M} \left(\boldsymbol{S}_j(n)\right)^2 \boldsymbol{a}(\theta_j)\boldsymbol{a}^H(\theta_j)$ is the interference signal covariance matrix ($\boldsymbol{R_i}$)

The output signal to interference plus noise ratio given as:

$$SINR_{in} = \frac{\boldsymbol{R_d}}{\boldsymbol{R_i} + \boldsymbol{R_n}} \quad (46)$$

Consider the output signal power of the beamformer; the output signal power is given as:

$$P_{out} = |\boldsymbol{y}(n)|^2 = E[\boldsymbol{y}(n)\boldsymbol{y}^H(n)] \quad (47)$$

But $\boldsymbol{y}(n) = \boldsymbol{w}^H\boldsymbol{X}(n)$, therefore substitute $\boldsymbol{y}(n) = \boldsymbol{w}^H\boldsymbol{X}(n)$, into the equation (3), we get:

$$P_{out} = E[(\boldsymbol{w}^H\boldsymbol{X}(n))(\boldsymbol{w}^H\boldsymbol{X}(n))^H] = \boldsymbol{w}^H E[\boldsymbol{X}(n)\boldsymbol{X}^H(n)]\boldsymbol{w} \quad (48)$$

Then $\boldsymbol{X}(n) = \boldsymbol{a}(\theta_d)\boldsymbol{S_d}(n) + \sum_{j=1}^{M} \boldsymbol{a}(\theta_j)\boldsymbol{S}_j(n) + \boldsymbol{N_o}(n)$,

therefore we get:

$$P_{out} = \boldsymbol{w}^H E\left[\left(\boldsymbol{a}(\theta_d)\boldsymbol{S_d}(n) + \sum_{j=1}^{M} \boldsymbol{a}(\theta_j)\boldsymbol{S}_j(n) + \boldsymbol{N_o}(n)\right)\left(\boldsymbol{a}(\theta_d)\boldsymbol{S_d}(n) + \sum_{j=1}^{M} \boldsymbol{a}(\theta_j)\boldsymbol{S}_j(n) + \boldsymbol{N_o}(n)\right)^H\right] w \quad (78)$$

$$= \boldsymbol{w}^H \left(\left(\boldsymbol{S_d}(n)\right)^2 \boldsymbol{a}(\theta_d)\boldsymbol{a}^H(\theta_d) \right) \boldsymbol{w} + \boldsymbol{w}^H \left(\sum_{j=1}^{M} \left(\boldsymbol{S}_j(n)\right)^2 \boldsymbol{a}(\theta_j)\boldsymbol{a}^H(\theta_j) \right) \boldsymbol{w} + \boldsymbol{w}^H \boldsymbol{N_o}(n)\boldsymbol{N_o}^H(n)\boldsymbol{w}$$

Where:

- $\boldsymbol{w}^H \left(\left(\boldsymbol{S_d}(n)\right)^2 \boldsymbol{a}(\theta_d)\boldsymbol{a}^H(\theta_d) \right) \boldsymbol{w}$ is output desired signal power (P_{d_out})
- $\boldsymbol{w}^H \left(\sum_{j=1}^{M} \left(\boldsymbol{S}_j(n)\right)^2 \boldsymbol{a}(\theta_j)\boldsymbol{a}^H(\theta_j) \right) \boldsymbol{w}$ is output interference signal power(P_{i_out})
- $\boldsymbol{w}^H \boldsymbol{N_o}(n)\boldsymbol{N_o}^H(n)\boldsymbol{w}$ is the output noise power (P_{n_out})

From the equation (4), we get:

$$SINR_{out} = \frac{\boldsymbol{w}^H \left(\left(\boldsymbol{S_d}(n)\right)^2 \boldsymbol{a}(\theta_d)\boldsymbol{a}^H(\theta_d) \right) \boldsymbol{w}}{\boldsymbol{w}^H \left(\sum_{j=1}^{M} \left(\boldsymbol{S}_j(n)\right)^2 \boldsymbol{a}(\theta_j)\boldsymbol{a}^H(\theta_j) \right) \boldsymbol{w} + \boldsymbol{w}^H \boldsymbol{N_o}(n).} \quad (49)$$

Where:

- $\left(\boldsymbol{S_d}(n)\right)^2 \boldsymbol{a}(\theta_d)\boldsymbol{a}^H(\theta_d)$ is the desired signal covariance matrix ($\boldsymbol{R_d}$)
- $\boldsymbol{N_o}(n)\boldsymbol{N_o}^H(n)$ is the noise covariance matrix ($\boldsymbol{R_n}$)
- $\sum_{j=1}^{M} \left(\boldsymbol{S}_j(n)\right)^2 \boldsymbol{a}(\theta_j)\boldsymbol{a}^H(\theta_j)$ is the interference signal covariance matrix ($\boldsymbol{R_i}$)

The output signal to interference plus noise ratio given as:

$$SINR_{out} = \frac{\boldsymbol{w}^H \boldsymbol{R_d}\, \boldsymbol{w}}{\boldsymbol{w}^H \boldsymbol{R_i}\, \boldsymbol{w} + \boldsymbol{w}^H \boldsymbol{R_n}\, \boldsymbol{w}} \quad (50)$$

But in this paper the interested SINR is the output SINR ($SINR_{out}$) aiming to know the performance at the output of the beamformer

4. SIMULATION RESULTS AND DISCUSSION

To verify the validity of this algorithm, the proposed solution was compared with modified LS-CMA in[1], conventional RLS-CMA and conventional LS-CMA. Assumption made: the sensor elements arranged in uniform linear array and space between each sensor is half wavelength. The signals received by the array of antenna is QPSK modulated desired signal from direction of $\theta = 0^{\circ}$ and two Gaussian interference signals from direction of $\theta = -40^{\circ}$ and $\theta = 40^{\circ}$. The simulation for this algorithm were taken under the following scenarios: 1) All signals have the same power 2) Desired signal power is much large power than interference signals and 3) Interference signals power are much large than desired signal

4.1. All signals have same power

The simulation under this condition is shown on the figures bellow;

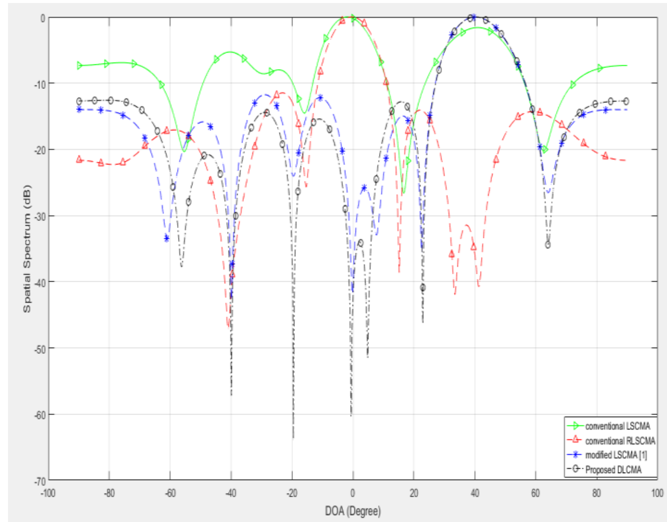

Figure 4 Beam pattern comparison. All signals having the same power

It seen that the proposed DL-CMA, modified LS-CMA in [1] and conventional RLSCMA steer the high power at the direction of arrival (DOA) of the QPSK modulated desired signal ($\theta = 0^{\circ}$) and suppress at the direction of arrival of interference signals ($\theta = -40^{\circ}$ and $\theta = 40^{\circ}$). Conventional LS-CMA steers the high power at the direction of arrival of interfering signal ($\theta = 0^{\circ}$and $\theta = -40^{\circ}$ and $\theta = 40$). The proposed DL-CMA has deep nulls at the direction of arrival of interference signals compared to others; therefore the proposed DL-CMA performs much better than modified LS-CMA in [11], conventional RLS-CMA and conventional LS-CMA.

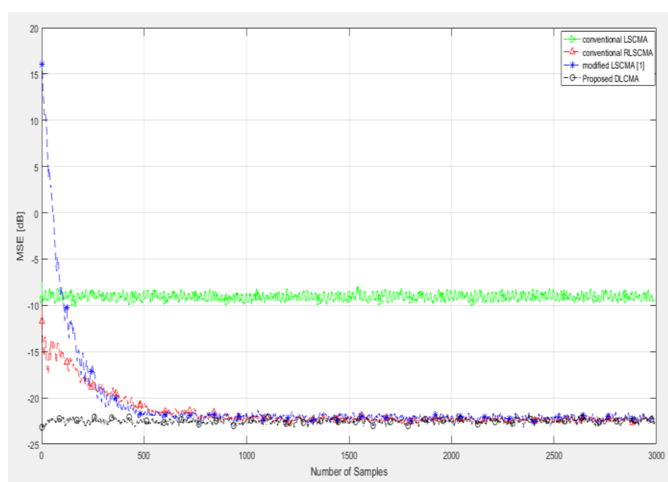

Figure 5 MSE comparison. All signals having the same power

Figure 5 shows that, the modified LS-CMA in[1], conventional RLS-CMA and conventional LS-CMA. This means that the performance of the proposed DL-CMA is better than modified LS-CMA in[1], conventional RLS-CMA and conventional LS-CMA

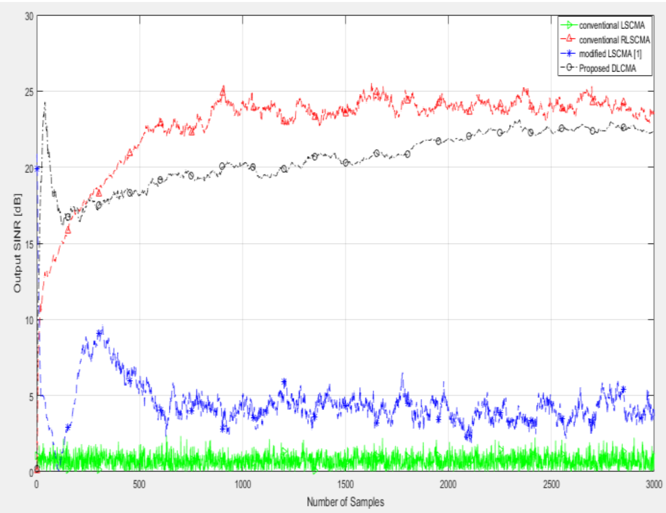

Figure 6 Output SINR comparison. All signals have the same power

Figure 6 shows that, the output SINR of the proposed DL-CMA increasing as the number of the sample increased. Also it shows that, the SINR of the proposed DL-CMA is higher than modified LS-CMA in[1], conventional RLS-CMA and conventional LS-CMA. This means that under this condition the performance of DL-CMA is good.

4.2. Desired signal power is much large than interference signals

The simulation under this condition is shown on the figures bellow;

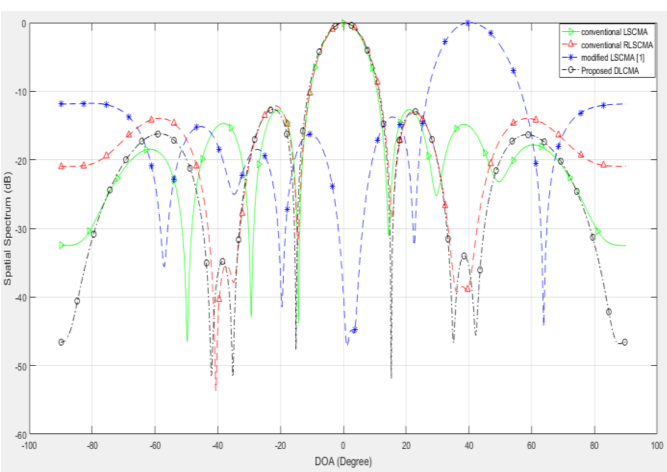

Figure 7 Beam pattern comparison. Desired signal power is much large than interference signals

Figure 7 shows that, that the proposed DL-CMA, modified LSCMA in[1], conventional RLS-CMA and conventional LS-CMA steer the high power at the direction of arrival (DOA) of the QPSK modulated desired signal ($\theta = 0^{\circ}$). Modified LS-CMA in[1], conventional RLS-CMA and conventional LS-CMA do not suppress the signal at the direction of arrival of interference signals ($\theta = -40^{\circ}$ and $\theta = 40^{\circ}$). The proposed DL-CMA suppresses the signal at the direction of arrival of interference signals. Therefore the proposed DL-CMA performs much better than modified LS-CMA in[1], conventional RLS-CMA and conventional LS-CMA.

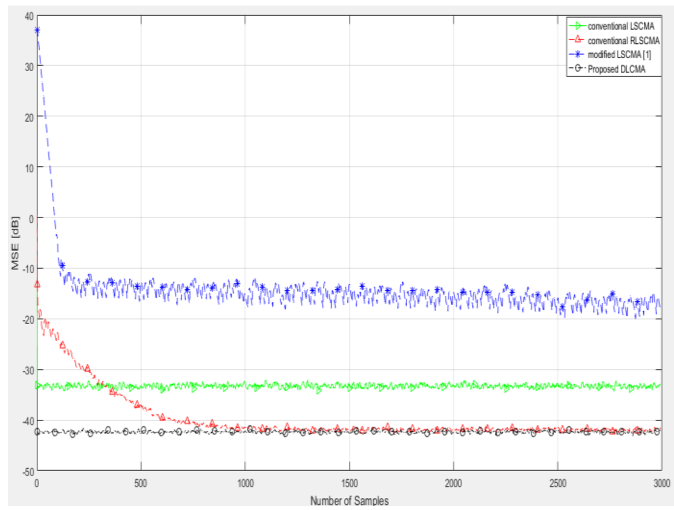

Figure 8 MSE comparison. Desired signal power is much large than interference signals

Figure 8 shows that, the proposed DL-CMA has a minimum MSE value compared to modified LSCMA in[1], conventional RLSCMA and conventional LS-CMA. This means that the performance of the proposed DL-CMA is better than modified LS-CMA in[1], conventional RLS-CMA and conventional LS-CMA.

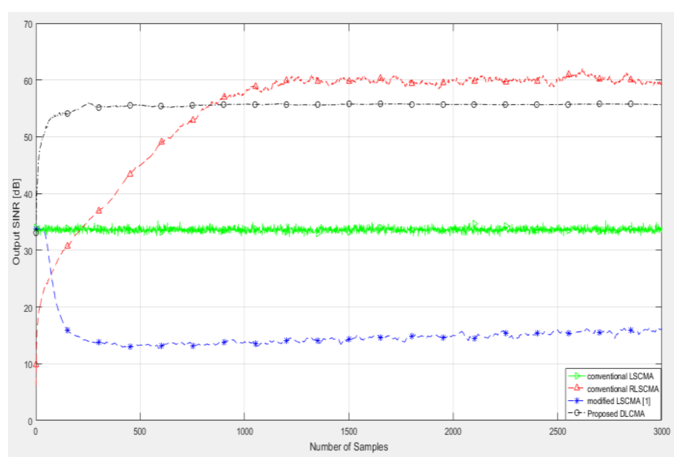

Figure 9 Output SINR comparison. Desired signal power is much large than interference signals

Figure 9 shows that, the SINR of the proposed DL-CMA is higher than modified LS-CMA in[1], conventional RLS-CMA and conventional LS-CMA. This means that under this condition the performance of DL-CMA is good.

4.3. Interference signals power are much large than desired signal

The simulation under this condition is shown on the figures bellow;

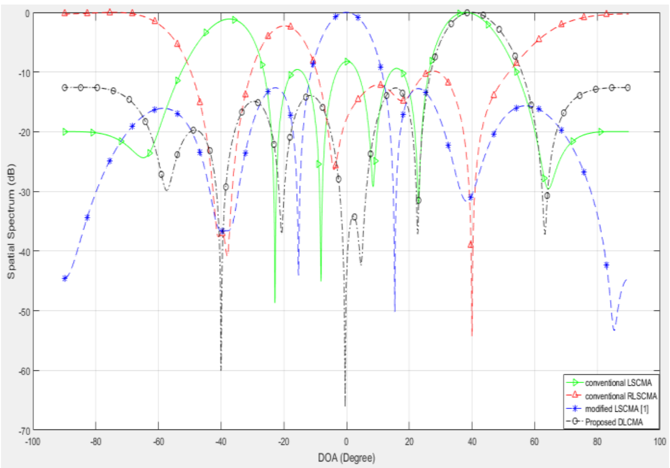

Figure 10 Beam pattern comparison. Interference signals power are much large than desired signal

Figure 10 shows that, the proposed DL-CMA, modified LS-CMA in [1] and conventional RLSCMA steer the high power at the direction of arrival (DOA) of the QPSK modulated desired signal ($\theta = 0^{\circ}$) and suppress at the direction of arrival of interference signals ($\theta = -40^{\circ}$ and $\theta = 40$). Conventional LS-CMA steers the high power at the direction of arrival of the interference signals ($\theta = -40^{\circ}$ and $\theta = 40$). The proposed DL-CMA has deep nulls at the direction of arrival of interfering signals compared to others; therefore the proposed DL-CMA performs much better than modified LS-CMA in[1], conventional RLS-CMA and conventional LS-CMA.

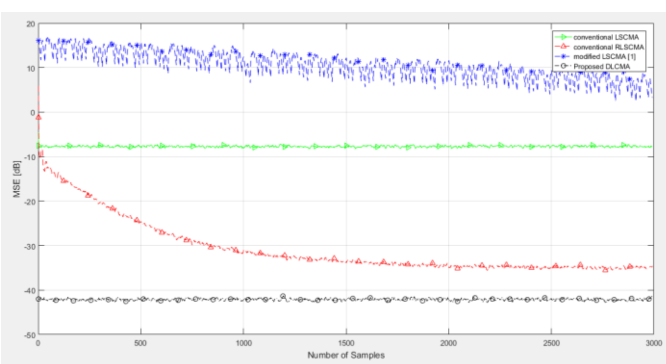

Figure 11 MSE comparison. Interference signals power are much large than desired signal

Figure 11 shows that, the proposed DL-CMA has a minimum MSE value compared to modified LS-CMA in[1], conventional RLS-CMA and conventional LS-CMA. This means that the performance of the proposed DL-CMA is better than modified LS-CMA in[1], conventional RLS-CMA and conventional LS-CMA

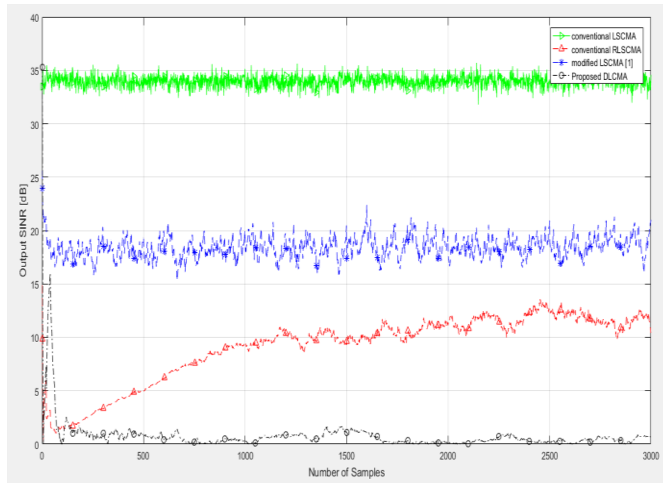

Figure 12 Output SINR comparison. Interference signals power are much large than desired signal

Figure 12 shows that, the output SINR of the proposed DL-CMA increasing as the number of the sample increased. Also it shows that, the SINR of the proposed DL-CMA is higher than modified LS-CMA in[1], conventional RLS-CMA and conventional LS-CMA. This means that under this condition the performance of DL-CMA is good.

5. CONCLUSION

In this paper, a new concept in in diagonal loading for robust beamforming was developed. The simulation results and analysis have been done by making comparison with the convectional RLS-CMA, conventional LS-CMA and modified LS-CMA[1] in order to ensure validity of the proposed method. This comparison has been done in terms of beam pattern, Mean Square error (MSE) value and, output SINR. The simulation results of the proposed method shows that, the method steers the main lobe at the direction of arrival of the desired signal and suppresses the interfering signals with the deep nulls. Also it has a better performance compared conventional LS-CMA and modified LS-CMA in[1]. Therefore, the proposed method solves the problem of steering the main lobe at the direction of arrival of interfering signals and hence improve the beamformer performance.

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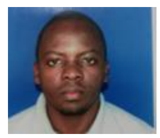

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